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NUMERICAL EXPERIMENT WITH A FRESNEL LENS

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Abstract—A mathematical formalism and a software package were developed with the purpose of investigating the characteristics of axially symmetric focusing optical elements. The computing experiment, described here in outline, yielded a detailed intensity distribution pattern in the focal spot of the focusator under study. The software takes into account the effects of discretization and quantization of the focusator phase function. The graphical performance of the integrated computer system is illustrated by distribution curves and a spatial distribution pattern for the Fresnel diffraction at a circular aperture and a Fresnel zone lens.

Production of plane optics elements (POE) involves a number of stages as follows [1–4]:

— computation of the POE transfer function on the basis of geometrical optics formulae,
— discretization of the POE for numerical analysis of its optical transmittance on a computer,
— a complicated manufacturing process that causes quantization of the phase component of the POE transfer function,
— simulation experiments on performance of the POE with the aid of a diffraction model.

To achieve any given POE performance specifications the designer has to investigate the diffraction effects associated with the technology of element design and manufacturing and to compensate for them. This problem may be reasonably considered as a computer experiment [3, 4] in view of the large amount of computational data involved, the considerable number of alternative parameters of the light field under study, the presence of a number of figures of merit, and the need to visualize the results. Being essentially a man–computer dialog, such an experiment calls for a fast diffraction analysis of the fine field structure at the POE focal diffraction spot, i.e. for appropriate algorithms and software packages.

The present paper is devoted to the methods and algorithms of diffraction analysis that underlie numerical experiments with axially symmetric focusing of laser light.

DIFFRACTION ANALYSIS FOR AXIALLY SYMMETRIC SYSTEMS

Let an axially symmetric optical system (Fig. 1) with optical transmission function $T(r)$ be illuminated by a light beam with complex amplitude $E(r) = A(r)\exp(i\psi(r))$, where $A(r)$ and $\psi(r)$ are the amplitude and phase of the incident wave. In view of the technology of POE design and manufacture, the optical transmission function of an axially symmetric element has the form [4, 5]

$$T(r) = \sum_{p=1}^{N} t_p \operatorname{rect}\left(\frac{r - r_p}{\delta_p} + \frac{1}{2}\right), \quad r \leqslant a, \tag{1}$$

where N is the number of POE resolved annular zones, a is the POE radius ($r_N = a$), and δ_p is the width of the pth zone with outer radius r_p and amplitude-phase transmission coefficient t_p.

Assuming $\delta = \min_p \delta_p \gg \lambda$ we resort to the Kirchhoff integral to estimate the complex amplitude $w(\rho, z)$ at the observation point (ρ, z), where $\rho = (x^2 + y^2)^{1/2}$. By virtue of Eq. (1), the complex amplitude at the observation point is represented as a superposition of the light fields due to the individual resolution annular zones (r_{p-1}, r_p), $p = 1, \ldots, N$,

$$w(\rho, z) = \sum_{p=1}^{N} t_p A(r_p - \delta_p/2)[w_p(\rho, z) - w_{p-1}(\rho, z)], \tag{2}$$

where $w_p(\rho, z)$ is the complex amplitude of the field due to a circular aperture of radius r_p. For

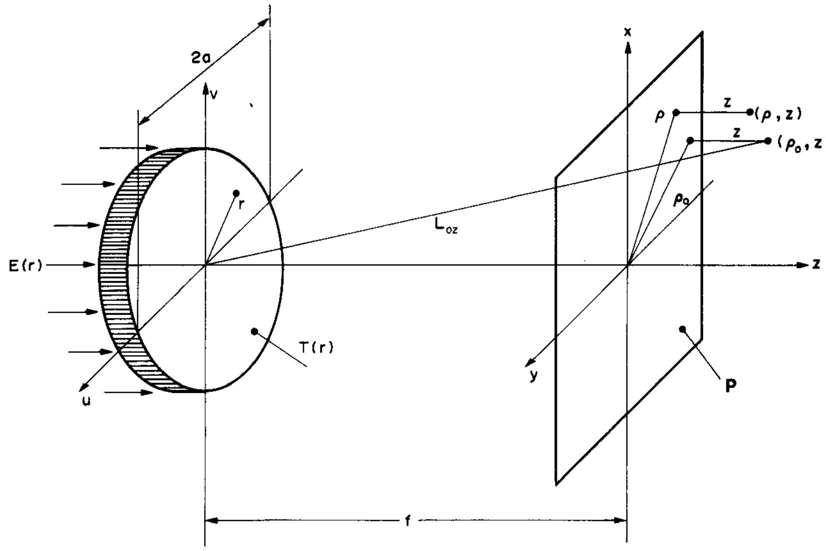

Fig. 1. Notation for an optical system with a plane optical element.

$2a/f \ll 1$, the complex field amplitude $w_p(\rho, z)$ at a point (ρ_0, z) may be computed as [4, 6]

$$w_p(\rho, z) = \frac{kr_p^2}{iL_{0z}q_p} \exp\left[ik\left(L_{0z} + \frac{\rho^2 - \rho_0^2}{2L_{0z}}\right)\right] E(q_p, s_p), \tag{3}$$

where

$$E(q_p, s_p) = \exp\left(-\frac{iq_p}{2}\right) U_1(q_p, s_p) + iU_2(q_p, s_p), \qquad \text{for} \quad |q_p/s_p| \leqslant 1,$$

$$= -i \exp\left(i\frac{s_p^2}{2q_p}\right) - \exp\left(-\frac{iq_p}{2}\right)[V_1(q_p, s_p) - iV_0(q_p, s_p)], \qquad \text{for} \quad |q_p/s_p| > 1.$$

Here, $L_{0z} = [(f+z)^2 + \rho_0^2]^{1/2}$, $q_p = kr_p^2(L_0^{-1} - L_{0z}^{-1})$, $s_p = kr_p\rho/L_{0z}$, and U_n and V_n are the Lommel functions [6], and $z = L_0 - f$ is the focusing point $(2a/L_0 \ll 1)$ of the beam $E(r)$ incident on the plane element.

This outlined computation of the diffraction integral is based on the fast computation of Lommel's function devised by Vasin *et al.* [2].

SOFTWARE PACKAGE FOR NUMERICAL EXPERIMENTATION

This software package has been developed on the basis of a program pack (PP) for image processing and digital holography [7]. It includes FORTRAN and PL/1 programs for calculating the diffraction integral and a module for connecting the program pack with the graphic programs GRAFOR [8].

The input data for the computation are sample arrays of the phase-amplitude transmittance of POE, discretization parameters, coordinates of observation points and the parameters of the optical system.

The output data are the arrays of intensity values and phase values of the light field recorded as the data base of the program package.

The experiment was carried out using a terminal of an ES-1055 computer equipped with an ES-7054 plotter, an intensified display, and a graphical display. The interactive process was based on simultaneous application of the program package, visualization facilities, computer graphic and the dialog regime. The intermediate experimental results were shown on the intensified display as patterns of intensity distribution. The graphic display was used to indicate the families of curves, isophotes, and 3-dimensional wavefield patterns. Hard copies could be obtained from the plotter, photoplotter and printer.

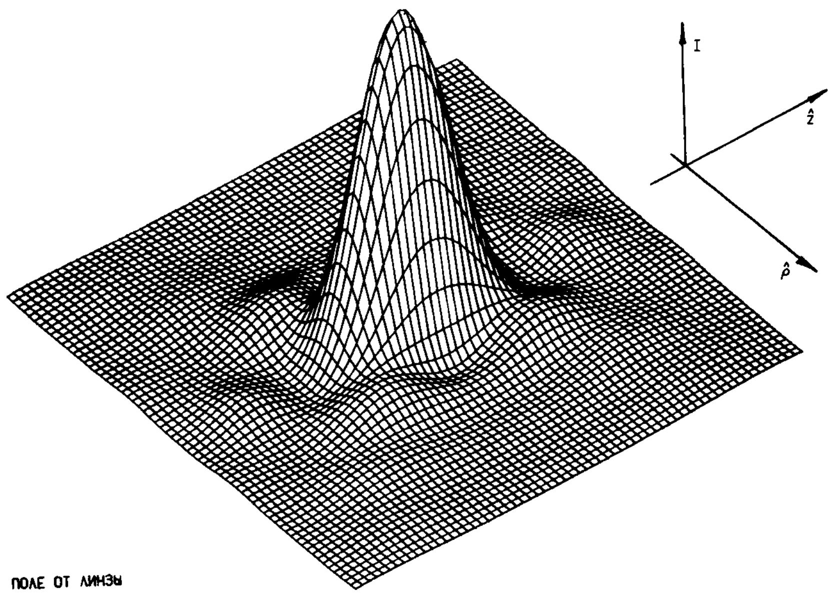

Fig. 2. Spatial intensity distribution in the focal domain of a lens ($0 \leqslant \hat{\rho} \leqslant 4\pi$, $-8\pi \leqslant \hat{z} \leqslant 8\pi$).

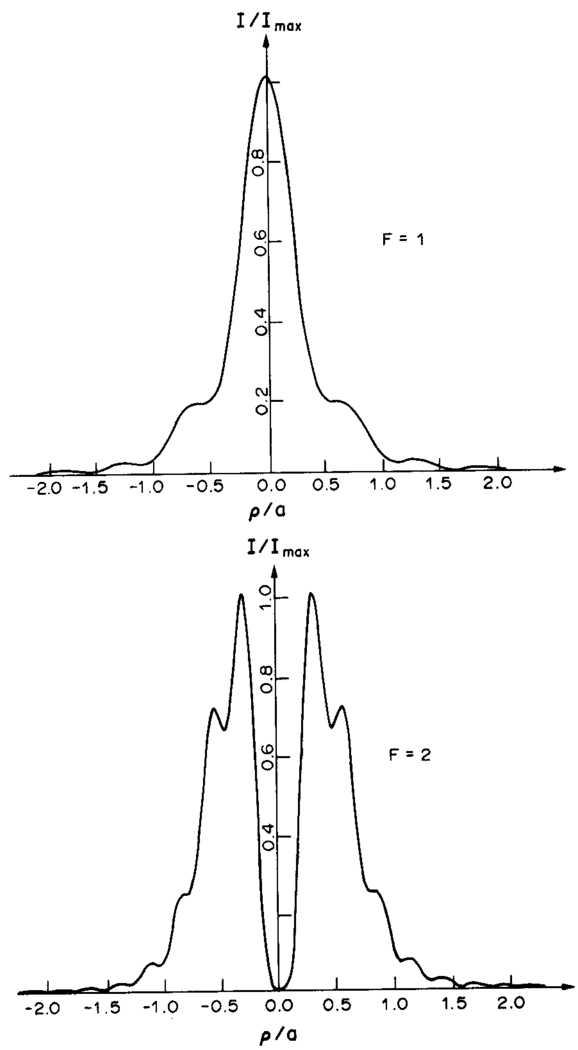

Fig. 3. Diffraction of a plane wave at a circular aperture. Intensity distributions for two Fresnel numbers $F = 1, 2$.

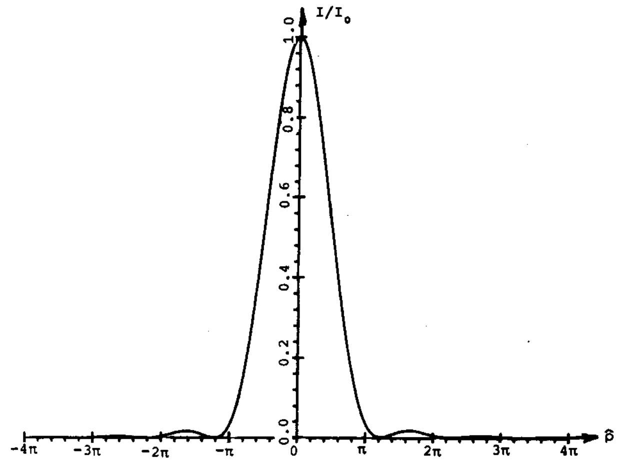

Fig. 4. Intensity distribution in the focal spot of a Fresnel lens.

ASSESSMENT OF THE NUMERICAL EXPERIMENT

Using the above program package we could readily reproduce the pattern given in Born and Wolf [6] for the case of diffraction of a converging spherical wave by a circular aperture. Moreover, the distribution was found to be asymmetric about the focal plane [9, 10] as is shown in Fig. 2 as a 3-dimensional pattern. In this figure, the vertical axis represents intensity, and the horizontal plane has dimensionless coordinates $\hat{\rho} = L_0\rho/f$ and $\hat{z} = ka^2z/(fL_{0z})$ that vary within the limits: $0 \leqslant \hat{\rho} \leqslant 4\pi$, $-8\pi \leqslant \hat{z} \leqslant 8\pi$.

At the same time, the method based on Lommel's functions fails to reveal a focal displacement of the intensity maximum towards the lens at small Fresnel numbers [9–12].

The accuracy of the Fresnel approximation used has been investigated by Southwell [13] and Goodman [14] who have demonstrated that for sufficiently large *f*-numbers ($f/2a > 12$) and $k \cdot L_{0z} \gg 1$, the computed modulus of the amplitude differs from the exact value by not more than 2%. This accuracy could be improved by incorporating into the experiment software package the methods based on direct calculation of the Kirchhoff integral [9, 10]. Such computations, however, take longer time and require more powerful computers.

Figure 3 plots the intensity distributions for various Fresnel numbers $F = a^2/\lambda f$ obtained for a plane wave diffracted by a circular aperture. These results agree completely with the findings in Kathuria [15] and Bottema [16] and allow us to study the diffraction of a plane light beam at optical elements with transmittance given by (1), say a Fresnel lens. We have reproduced the results of Bottema [16] for a Fresnel zone plate and plotted the intensity distribution in the principal focal plane of a phase lens (Fig. 4). The efficiency of a phase Fresnel lens was estimated to be $4/\pi^2$ of a perfect lens with the same parameters.

CONCLUSION

A numerical experiment with a lens and a Fresnel lens has demonstrated adequate performance of the software package developed for this experiment and borne out that the underlying algorithms and computational techniques are correct. These results pave the way for the study of complex axially symmetric focusers of laser radiation.

REFERENCES

1. I. N. Sisakyan and V. A. Soifer. *Physical Fundamentals and Applications of Holography*. LIYaF Press, Leningrad (1984).
2. A. G. Vasin, M. A. Golub, V. A. Danilov, N. L. Kazansky, S. V. Karpeyev, I. N. Sisakyan, V. A. Soifer and F. V. Uvarov. *Preprint FIAN SSSR* (Physical Institute of the U.S.S.R. Academy of Sciences), No. 304, Moscow (1983).
3. V. A. Soifer. *Current Problems of Physics and its Applications*, Pt. 1, p. 5. VINITI Press, Moscow (1987).
4. M. A. Golub, N. L. Kazansky, I. N. Sisakyan and V. A. Soifer. *Avtometriya*, No. 5 (1987).
5. S. T. Bobrov, G. I. Greisukh and Yu. G. Turkevich. *Optics of Diffraction elements and Systems*. Mashinostroyeniye, Leningrad (1986).

6. M. Born and E. Wolf. *Principles of Optics*, 4th edn. Pergamon Press, Oxford (1970).
7. Application program package for image processing and digital holography. The State Pool of Algorithms and Programs. Reg. No. P004582 dated 01.09.1980.
8. Yu. M. Bayakovsky, A. V. Galaktionov and T. P. Mikhailova. *GRAFOR—A Graphic Expansion of FORTRAN*. Nauka, Moscow (1985).
9. B. E. Kinber and S. V. Novoselov. *Radiotekh. Elektron.* **30**, 1469 (1985).
10. Y. Li and E. Wolf. *J. Opt. Soc. Am.* **AI**, 801 (1984).
11. A. S. Dementyev and D. P. Domarkene. *Opt. Spektrosk.* **56**, 869 (1984).
12. Y. Li and J. S. Horng. *Optik* **70**, 127 (1985).
13. W. H. Southwell. *J. Opt. Soc. Am.* **71**, 7 (1981).
14. D. Goodman. *J. Opt. Soc. Am.* **72**, 1560 (1982).
15. Y. P. Kathuria. *Optica Applicata* **14**, 509 (1984).
16. M. Bottema. *J. Opt. Soc. Am.* **59**, 1632 (1969).